

Measure and collapse of participatory democracy in a two party system.

Jozef Sznajd

*Institute for Low Temperature and Structure Research,
Polish Academy of Sciences, Wroclaw*

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Abstract

Measure what is measurable, and make measurable what is not so (Galileo Galilei). According to the above sentence we do not ask why we need to measure democracy but if it is possible to measure something which is not unequivocally defined. Although, it is unlikely a final agreement on the definition of democracy, the idea that it is a form of governance based on collective decision making seems to be uncontested. On the premise that in the high-quality democracy citizens (agents) not only must have equal participation rights but *must want to participate in shaping decision*, as an effective measure of democracy in a two party political system we propose the percentage of the total population that actually voted in a given elections only for two major parties. Thus, we disregard not only nonvoters but also smaller parties voters whom votes will not have a substantial impact on the election and consequently they will not be in the loop, even theoretically. To describe such a system a sociophysics model based on the $S = 1$ Ising model (Blume-Capel) is proposed. The measure of democracy, V_D index, as a function of inter-party conflict is analyzed.

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I. INTRODUCTION

Over half of the world's countries can be considered democracies. However, the quality of democracy in particular countries may be quite different. The question is if there is a way to distinguish the quality of democracy or in other words if there is a sensible measure of democracy. There is a number available measures - indices such as: Democracy Index, Freedom House, Polity, Democracy Barometer or Vanhanen Index which try to take into account various aspects of democracy and consequently are based on many indicators and subjective assessments.

The democracy index is based on 60 indicators grouped in five categories: electoral process, civil liberties, political participation and culture, and ranks countries as one of types: "full democracies", "flawed democracies", "hybrid regimes" and "authoritarian regimes" (incidentally, from a physicist point of view these categories resemble the 18th century definitions of the Fahrenheit scale points: aestus intolerabilis (blinding heat), calor ingens (great heat), aer temperatus (moderate), aer frigidus (cold), ...) . *Freedom House* assesses the current state of political rights and civil liberties in each state on a scale from 1 (most free) to 7 (least free) and then classified as "free", "partly free", or "not free". *Polity's* conclusions are based on an evaluation of an election, the nature of political participation, and the extent of checks on executive authority. The Polity scale ranges from -10 to 10 from "autocracies" (-10 to -6), through anocracies (-5 to 5) to democracies (6 to 10). *The Democracy Barometer*¹ is based on the idea that one can measure the degree of fulfillment of the nine "functions" deduced from three principles: Freedom (functions: Individual liberties, Rule of Law, Public Sphere), Control (Competition, Mutual Constraints, Governmental Capability), and Equality (Transparency, Participation, Representation). DB consists of a total of 100 indicators. *The Vanhanen Index*² is based on two clearly defined quantitative indicators corresponding to the two theoretical dimensions of democratization called: "competition" and "participation". According to the Vanhanen idea the "degree of competition" in a given political system is indicated by the electoral success of the smaller parties, and the "degree of electoral participation" is measured by the percentage of the total population that actually voted in a given elections. These two variables are taken with the same weight to construct an index of democratization (ID) - Vanhanen Index.

None of the above mentioned indices has received common acceptance, and except for

the Vanhanen Index their construction is rather complicated and linked to a certain extent with policy. So, it seems to be also helpful to analyze the problem of measure of democracy by using statistical physics models or sociophysics approach³⁻⁵.

II. THE MODEL

The starting point is the premise that in the high-quality democracy citizens (agents) not only must have equal participation rights, which is obvious, but also *must want to participate in shaping decision*. In this paper we confine ourselves to consider two party system i.e. political system in which the electorate votes mostly only for two major parties. So, one or the other party can win a majority in the legislature. In consequence votes given to smaller parties have only formal meaning without a real impact on shaping decision. The classical example of a state with the two party system is of course the U.S. where in fact all members of the parliament belong to one of the two major parties. However, more common is the two party system where two major parties dominate elections but there are third parties which have some seats in the legislature. The examples are the United Kingdom or Poland for eight years.

As an effective measure of democracy in the two party political system we propose the percentage of the total population that actually voted for two major parties in a given elections. Thus we divide the whole population entitled to vote into three groups: the electorate of the first party called L , the electorate of the second party called C , and the others called F . The latter group form: the smaller (third parties) voters which in the main vote "against" and are fully aware that their voices will not have a major impact on the practical outcome of the election, floating voters or indifferent citizens. So, the effective democracy measure V_D is given by

$$V_D = \frac{N_L + N_C}{N_L + N_C + N_F}, \quad (1)$$

where N_L, N_C, N_F denote numbers of the particular parties voters.

According to the sociophysics idea social behavior can be modeled in the same way that physics models natural phenomena³. The most popular and useful physics model applied to describe social behavior is undoubtedly the Ising model^{3-5,7,8}. So, in the sociophysics language we consider a group of n agents (citizens), where $n = n_L + n_C + n_F$, and n_L, n_C, n_F

denote initial numbers of the "L", "C", and "F" party voters, respectively. Each of the agent has attached an Ising variable (spin) S_i^α , where $\alpha = L, C, F$ and $i = 1, 2, \dots, n_\alpha$. In this case the Ising variable has three possible values, and when the agent " i " is a voter of "L" party, we take $S_i = 1$, when the agent is "C"-voter, $S_i = -1$, and for "F"-voter, $S_i = 0$. Analogously, as in the physics case we introduce coupling between two agents from the same group $K_\alpha (\alpha = L, C, F)$ which is a measure of the unity of views or satisfaction to be a member of the same group. In a stable situation the coupling K_F is negligible because usually the members of the "F" party have no common views. To distinguish creeds of the electorates of the "L" and "C" parties we introduce an external field $H_\beta (\beta = L, C)$ coupled linearly with each agent of the "L" and "C" groups. The members of the "F" group are not able to distinguish between "L" and "C" party, so their "creed" has to be independent of the sign $+$, $-$, i.e. a "field" D should be coupled to $(S_i^F)^2$. Finally, confining ourselves to the one dimensional arrangement of the particular subgroup members, one has three decoupled chains described by the Hamiltonian:

$$\tilde{H}_0 = - \sum_{\alpha=L,C,F} K_\alpha \sum_{i=1}^{n_\alpha} S_i^\alpha S_{i+1}^\alpha - \sum_{\beta=L,C} H_\beta \sum_{i=1}^{n_\beta} S_i^\beta - D_F \sum_{i=1}^{n_F} (S_i^F)^2. \quad (2)$$

Postulating a principle of maximum satisfaction³ one can find the equilibrium state of the model described by the Hamiltonian (2). And if

$$\text{sgn}(H_L) = -\text{sgn}(H_C), \quad K_\alpha > 0, \quad \text{and} \quad D_F < -K_F, \quad (3)$$

then, for the isolated system at the counterpart of the physical ground state (zero temperature) all agents of the "L" group have spin $+1$, all agents of the "C" group have spin -1 , and all members of the "F" group have spin 0 . In this paper we consider only equilibrium properties of the system which in physics depend on temperature. In principle such a quantity does not exist in social system. However, there is the social meaning of temperature T in sociophysics as an overall approximation for all random events which influence decisions but are not included in the model⁶. Accordingly, one can assume that social systems have their "temperature" at their steady state which validates an application of the finite temperature statistical physics methods to study social systems. Whereas, at $T = 0$ all agents from "L" group have spin $+1$, from "C" group -1 , and from "F" group 0 , at finite temperature only N_α^+ members of " α " group have still spin $+1$, N_α^- spin -1 , and N_α^0 spin 0 . Consequently,

at finite temperature "α" party has N_α voters ($\alpha = L, C, F$)

$$N_L = \sum_{\alpha=L,C,F} N_\alpha^+, \quad N_C = \sum_{\alpha=L,C,F} N_\alpha^-, \quad N_F = \sum_{\alpha=L,C,F} N_\alpha^0 \quad (4)$$

and the quantities N_α^χ ($\chi = +, -, 0$) expressed by the spin averages in the following way:

$$N_\alpha^+ = \frac{1}{2}(\langle S_\alpha^2 \rangle + \langle S_\alpha \rangle), \quad N_\alpha^- = \frac{1}{2}(\langle S_\alpha^2 \rangle - \langle S_\alpha \rangle), \quad N_\alpha^0 = 1 - N_\alpha^+ - N_\alpha^-. \quad (5)$$

As a measure of political strife between electorates of the two major parties "L" and "C" we introduce a coupling Q

$$-Q \sum_i (S_i^L)^2 (S_i^C)^2. \quad (6)$$

The choice of such a coupling prefers an exchange of voters between the "L" (or "C") and "F" group rather than between "L" and "C" which is possible but less probable from the ideological point of view.

III. THE METHOD

The obvious way to analyze flows of the voters between the parties are computer simulations. However, in this paper we concentrate on the equilibrium properties using to study the Hamiltonian (2,6) the linear renormalization group transformation. We start with the three decoupled chains (2), assuming that initially the number of the voters in each group (L, C, F) is the same $n_\alpha = n$, and

$$H_0 = -\beta \tilde{H}_0 = \sum_{\alpha=L,C,F} H_0^\alpha, \quad H_0^\alpha = k_\alpha \sum_{i=1}^n S_i^\alpha S_{i+1}^\alpha + h_\alpha \sum_{i=1}^n S_i^\alpha + d_\alpha \sum_{i=1}^n (S_i^\alpha)^2, \quad (7)$$

where $k_\alpha = -K_\alpha/T$, $h_\alpha = -H_\alpha/T$, $d_\alpha = -D_\alpha/T$. The minimal set of the parameters to describe our model consists three intrachain couplings $k = k_L = k_C$, $h = h_L = -h_C$, and $d = d_F$ and yields

$$H_0^L = k \sum_{i=1}^n S_i^L S_{i+1}^L + h \sum_{i=1}^n S_i^L, \quad H_0^C = k \sum_{i=1}^n S_i^C S_{i+1}^C - h \sum_{i=1}^n S_i^C, \quad H_0^F = d \sum_{i=1}^n (S_i^F)^2. \quad (8)$$

The renormalization group transformation for the Hamiltonian (7) is defined by

$$\exp[H'_0(\sigma)] = \text{Tr}_S P(\sigma, S) \exp[H_0(S)], \quad (9)$$

and the weight operator $P(\sigma, S)$ which couples the original S and effective σ spins is chosen in the linear form⁹

$$P(\sigma, S) = \prod_i p_i = \prod_i (1 - S_{2i+1}^2 - \sigma_{i+1}^2 + \frac{1}{2} S_{2i+1} \sigma_{i+1} + \frac{3}{2} S_{2i+1}^2 \sigma_{i+1}^2) \quad (10)$$

For the decoupled chains the transformation (9, 10) is a decimation transformation where in each step of the procedure every other spin is killed and the renormalized Hamiltonian can be written in the form

$$H'(\sigma^\alpha) = \sum_{\alpha=L,C,F} \ln Tr_{S^\alpha} e^{H_0^\alpha(S^\alpha)} \quad (11)$$

Unlike the case of the two-state model ($S = 1/2$ Ising model), the decimation transformation for three-state ($S = 1$) model generates new interactions

$$j_\alpha(S_i^\alpha)^2 S_{i+1}^\alpha, \quad q_\alpha(S_i^\alpha)^2 (S_{i+1}^\alpha)^2, \quad h_F S_i^F, \quad d_L (S_i^L)^2, \quad d_C (S_i^C)^2, \quad (12)$$

and finally

$$\begin{aligned} \ln Tr_{S^\alpha} e^{H_0^\alpha(S^\alpha)} &= \ln [f_0^\alpha + f_1^\alpha \sigma_i^\alpha + f_2^\alpha \sigma_i^\alpha \sigma_{i+1}^\alpha + f_3^\alpha (\sigma_i^\alpha)^2 + f_4^\alpha (\sigma_i^\alpha)^2 \sigma_{i+1}^\alpha + f_5^\alpha (\sigma_i^\alpha \sigma_{i+1}^\alpha)^2] \\ &= z_\alpha + h'_\alpha \sigma_i^\alpha + k'_\alpha \sigma_i^\alpha \sigma_{i+1}^\alpha + d'_\alpha (\sigma_i^\alpha)^2 + j'_\alpha (\sigma_i^\alpha)^2 \sigma_{i+1}^\alpha + q'_\alpha (\sigma_i^\alpha \sigma_{i+1}^\alpha)^2. \end{aligned} \quad (13)$$

The renormalized parameters $h'_\alpha, k'_\alpha, d'_\alpha, j'_\alpha, q'_\alpha$ and z_α as functions of the original interactions are presented in the Appendix A. The constant term z_α (independent of effective spins σ) can be used to calculate the "free energy" per site

$$f = \frac{1}{3} \sum_{n=1}^{\infty} \frac{z_L^{(n)} + z_C^{(n)} + z_F^{(n)}}{2^n}, \quad (14)$$

where n numbers the RG steps, and hence the spin averages $\langle S^\alpha \rangle$ and $\langle (S^\alpha)^2 \rangle$. In Fig.1 the temperature dependences of the spin averages found from the RG procedure for infinite chains (solid lines), and exact results for three three-site chains (dashed lines) are presented for the model with

$$D_F = -1.1, \quad K_L = K_C = 0.5, \quad H_L = -H_C = 0.01. \quad (15)$$

As seen, the results for the infinite and three site chains converge at $T = 0$ and for high temperatures.

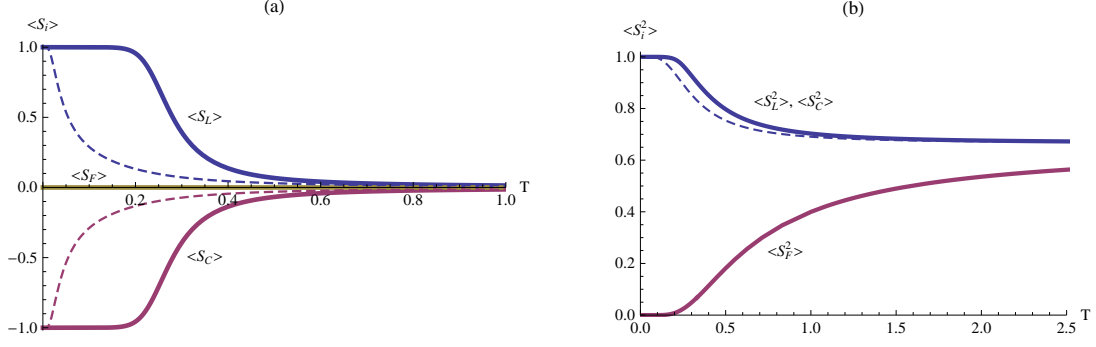


FIG. 1: Temperature dependence of the magnetization $\langle S_\alpha \rangle$ (a) and $\langle S_\alpha^2 \rangle$ (b) for three noninteracting chains. Solid lines denote infinite chains and dashed lines three-spin chains.

In order to consider the interchain (intergroup) coupling (6) we apply a cluster approximation. In this approximation one considers a finite number of isolated cells (cluster) disregarding the remaining cells of the system¹⁰. Outwardly, in our case the simplest cluster possible is made of two three-site cells from "L" and "C" subsystems and the contribution to the renormalized energy of this cluster is

$$\ln \langle e^{\sum_{i=1}^3 q(S_i^L)^2(S_i^C)^2} \rangle_0, \quad q = q_{LC} = -Q/T, \quad (16)$$

where

$$\langle A \rangle_0 = \frac{\text{Tr}_S A P(\sigma, S) e_0^H}{\text{Tr}_S P(\sigma, S) e_0^H}. \quad (17)$$

However, as usual the RG procedure generates new couplings, whose original values are equal to zero, and one has to consider general interaction of the isolated set of the three three-site cells from "L", "C", and "F" subsystems

$$\ln \langle e^{H_I} \rangle \quad (18)$$

with

$$\begin{aligned} H_I = & \sum_{\alpha \neq \beta = L, C, F} k_{\alpha\beta} \sum_{i=1}^3 S_i^\alpha S_i^\beta + \sum_{\alpha \neq \beta = L, C, F} q_{\alpha\beta} \sum_{i=1}^3 (S_i^\alpha S_i^\beta)^2 + \sum_{\alpha \neq \beta = L, C, F} j_{\alpha\beta} \sum_{i=1}^3 (S_i^\alpha)^2 S_i^\beta \quad (19) \\ & + \sum_{\alpha \neq \beta = L, C, F} k_{\alpha\beta}^d \sum_{i=1}^3 S_i^\alpha S_{i+1}^\beta + \sum_{\alpha \neq \beta = L, C, F} q_{\alpha\beta}^d \sum_{i=1}^3 (S_i^\alpha S_{i+1}^\beta)^2 + \sum_{\alpha \neq \beta = L, C, F} j_{\alpha\beta}^d \sum_{i=1}^3 (S_i^\alpha)^2 S_{i+1}^\beta \\ & + \sum_{\alpha \neq \beta = L, C, F} j_{\beta\alpha} \sum_{i=1}^3 (S_i^\beta)^2 S_i^\alpha + \sum_{\alpha \neq \beta = L, C, F} j_{\beta\alpha} \sum_{i=1}^3 (S_i^\beta)^2 S_{i+1}^\alpha. \end{aligned}$$

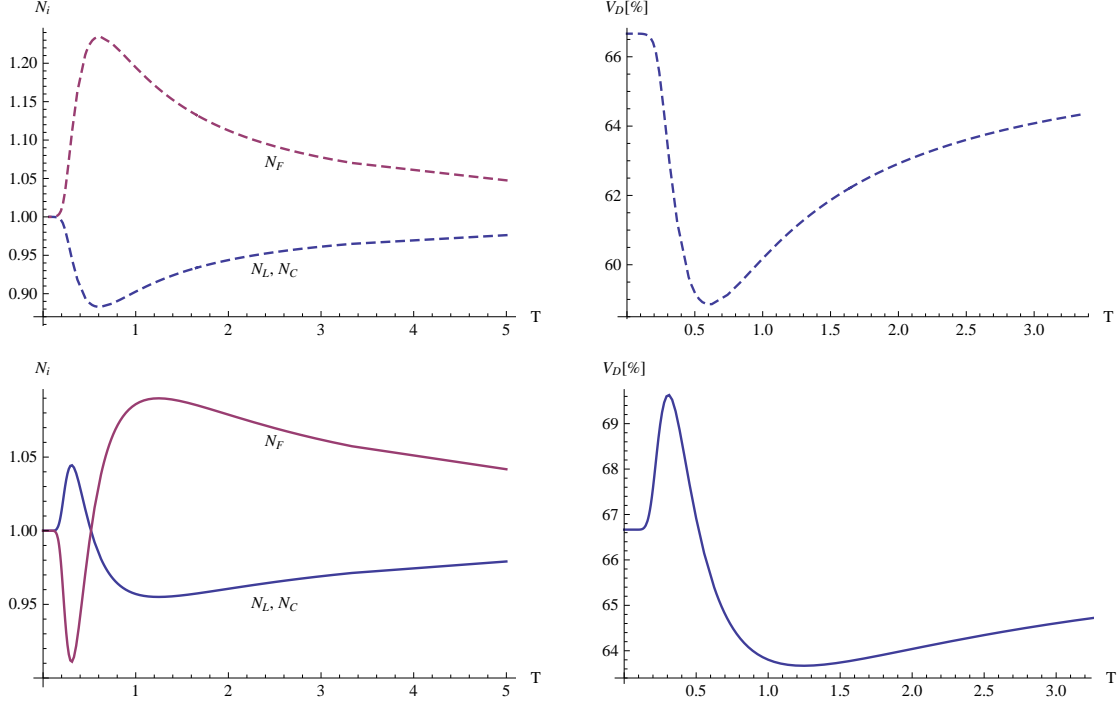


FIG. 2: The temperature dependences of the voter numbers N_i per agent and index V_D (in percent) for $K_C = K_L = 0.5, K_F = 0$ (dashed lines) and $K_C = K_L = 0.5, K_F = 1$, (solid lines).

Altogether, one has to consider, formally, 39 coupling parameters 15 single-chain (3 chains times 5 parameters) in that, in the minimum set, three original k, h, d (Eq.8) and 24 interchain couplings (Eq.19) in that one original q (Eq.16). However, it is quite easy to find analytical forms of the renormalized couplings and perform the RG iterations.

IV. THE V_D INDEX

As mentioned in the Introduction the random events but also an information noise can play in social systems a similar role as temperature in physical systems. So, one can find the temperature dependences of the number of particular party voters and V_D . We start with the noninteracting subsystems models: (i) defined by the parameters (15) and to check the possible role of the K_F coupling (ii) additionally $K_F = 1$. Because we assume that numbers of each group voters are equal to each others $n_L = n_C = n_F$, hence initially (at the ground state) the numbers of each group "L", "C" and "F" voters are the same $N_L = N_C = N_F = 1$ per agent.

The results are presented in Fig.2. As seen for model (i) the voter numbers start with

$N_i = 1$ and then with the temperature increasing, N_L and N_C decrease, reach a minimum and then increase to 1 at $T \rightarrow \infty$. Similarly, the V_D -index starts with $\frac{2}{3}$ at $T = 0$ and passing the minimum reaches the same value $\frac{2}{3}$ at $T \rightarrow \infty$. For the model (ii) both $N_L(N_C)$ and V_D first increase, reach the maxima, pass through a minima, and then increase again to 1 and $\frac{2}{3}$ at $T \rightarrow \infty$, respectively.

We are now in a position to evaluate the dependence of the particular parties voter numbers on the intergroup coupling Q (6, 16). Let us first consider the finite system of three chains of three agents in each of them with the coupling parameters as in (15) and $Q > 0$ at $T = 0$ (ground state), $T = 0.05$, and $T = 0.2$. As is seen from the left plots of Fig.3 at the ground state the initial agent arrangement $N_L = N_C = N_F = 1$ is conserved until $Q = \frac{1}{3}$, then there is a jump of $N_L = N_C$ to $\frac{1}{2}$, and N_F to 2. The jump is gradually smeared by rising temperature (middle and right plots of Fig.3). For a nonsymmetric case $K_L \neq K_C$ (Fig.4), the ground state configuration is essentially different and if, for example $K_L > K_C$ then N_L does not depend on Q whereas N_C drops to 0 and N_F jumps to 2 at $Q = \frac{1}{3}$. At higher temperature the behavior of N_i in the nonsymmetric case is similar to that of symmetric one. In the bottom plots of Figs.3 and 4 the Q -dependence of the V_D index is shown. For low temperature V_D drops sharply at $Q = \frac{1}{3}$, and for higher temperature it decreases rapidly from $\frac{2}{3}$ to $\frac{1}{3}$.

Now we proceed to the RG analysis of the infinite chains. To calculate the average (18) we use the identity

$$\begin{aligned} \exp[k_{\alpha\beta} S^\alpha S^\beta + q_{\alpha\beta} (S^\alpha S^\beta)^2 + j_{\alpha\beta} (S^\alpha)^2 S^\beta + j_{\beta\alpha} S^\alpha (S^\beta)^2] = \\ 1 + K_{\alpha\beta} S^\alpha S^\beta + Q_{\alpha\beta} (S^\alpha S^\beta)^2 + J_{\alpha\beta} (S^\alpha)^2 S^\beta + J_{\beta\alpha} S^\alpha (S^\beta)^2, \end{aligned} \quad (20)$$

where

$$\begin{aligned} K_{\alpha\beta} &= \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} - j_{\alpha\beta} - j_{\beta\alpha}} (e^{2k_{\alpha\beta}} - e^{2j_{\alpha\beta}} - e^{2k_{\beta\alpha} + 2j_{\alpha\beta} + 2j_{\beta\alpha}}), \\ Q_{\alpha\beta} &= -1 + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} - j_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} - j_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} + j_{\alpha\beta} + j_{\beta\alpha}}, \\ J_{\alpha\beta} &= -\frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} - j_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} - j_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} + j_{\alpha\beta} + j_{\beta\alpha}}, \\ J_{\beta\alpha} &= -\frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} - j_{\alpha\beta} - j_{\beta\alpha}} - \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} + j_{\alpha\beta} - j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} - k_{\alpha\beta} - j_{\alpha\beta} + j_{\beta\alpha}} + \frac{1}{4} e^{q_{\alpha\beta} + k_{\alpha\beta} + j_{\alpha\beta} + j_{\beta\alpha}}. \end{aligned} \quad (21)$$

To evaluate the RG transformation one has to know the chain averages $\langle S_i^\alpha \rangle$, $\langle (S_i^\alpha)^2 \rangle$, $\langle S_i^\alpha S_i^\beta \rangle$, $\langle (S_i^\alpha S_i^\beta)^2 \rangle$, and $\langle (S_i^\alpha)^2 S_i^\beta \rangle$. It is quite easy to find their closed expressions

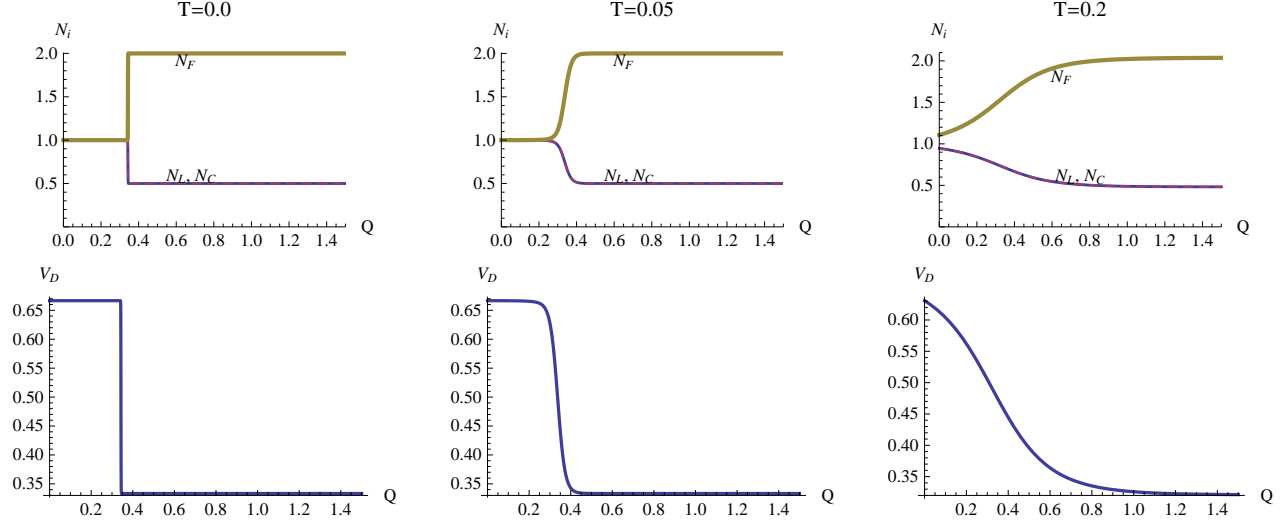


FIG. 3: Finite system: Q -dependence of N_i for $K_L = K_C = 0.5$ and several temperatures.

and for example:

$$\begin{aligned}
 \langle S_1^\alpha \rangle &= \sigma_1^\alpha, \quad \langle S_3^\alpha \rangle = \sigma_2^\alpha, \quad \alpha = L, C, F \\
 \langle S_2^\alpha \rangle &= G_0^\alpha + G_1^\alpha(\sigma_1^\alpha + \sigma_2^\alpha) + g_g^\alpha \sigma_1^\alpha \sigma_2^\alpha + G_2^\alpha[(\sigma_1^\alpha)^2 + (\sigma_2^\alpha)^2] \\
 &\quad + G_3^\alpha[(\sigma_1^\alpha)^2 \sigma_2^\alpha + (\sigma_2^\alpha)^2 \sigma_1^\alpha] + G_4^\alpha(\sigma_1^\alpha)^2 (\sigma_2^\alpha)^2.
 \end{aligned} \tag{22}$$

The coefficients G_i^α are presented in Appendix B.

In the cluster approximation with three three-site (-agent) blocks "L", "C", "F" taking into account only two-site coupling, the RG transformation has the form of 39 recursion relations. Iterating these relations and collecting the constant terms generated in each step of the iteration process one can calculate numerically the "free energy" and then the averages $\langle S_i^\alpha \rangle$ and $\langle (S_i^\alpha)^2 \rangle$. In Fig.5 these averages are presented as functions of interblock coupling Q for the model with $D_F = -1.1, H_L = -H_C = 0.01$ in two cases: (i) symmetric $K_L = K_C = 0.5$ and (ii) nonsymmetric $K_L = 0.5$ and $K_C = 0.48$ at $T = 0.25$. Knowing the averages $\langle S_i^\alpha \rangle$ and $\langle (S_i^\alpha)^2 \rangle$ one can find the number of particular parties voters (5) and V_D index (1). The results for symmetric and nonsymmetric cases are presented in Figs. 6 and 7, respectively. As seen the dependences of the voter numbers on Q for infinite system differ significantly from those for three-site blocks. However, in both cases symmetric and nonsymmetric as for the finite system at low temperature the V_D index changes slowly for sufficiently small Q and then drop sharply to a constant value.

In physical systems the coupling parameters $K_\alpha, D_\alpha, H_\alpha$ or Q_α have plausible interpreta-

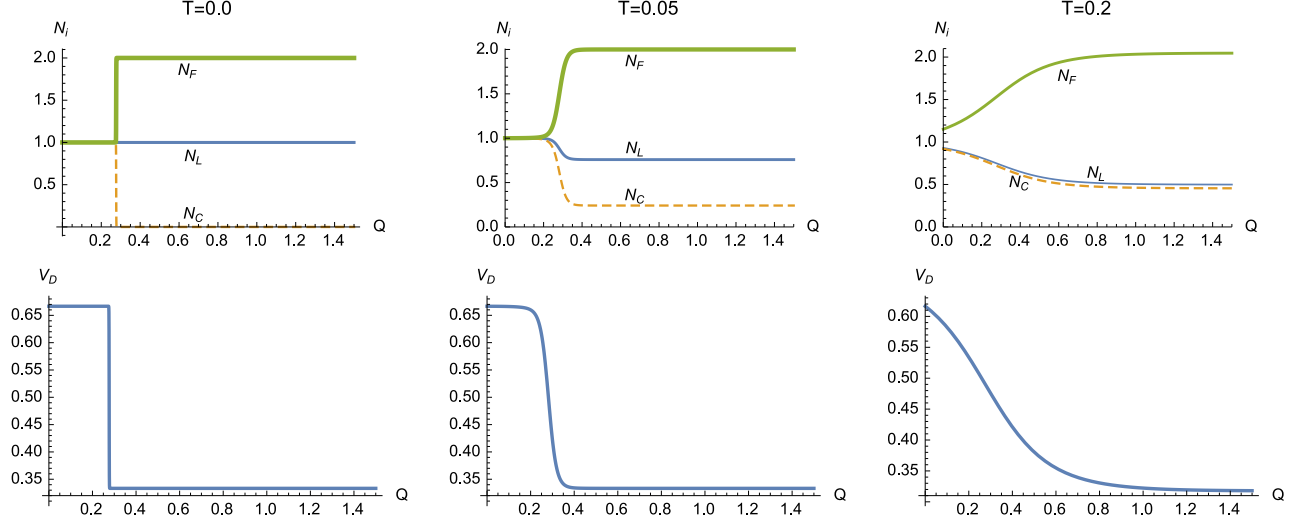


FIG. 4: Finite system: Q -dependence of N_i for $K_L = 0.5, K_C = 0.48$ and several temperatures.

tion even if they have an effective character. Such an interpretation is not of course so obvious for social systems. However, one can assume that there is a positive coupling between the members of the same political environment which measure is the parameter K_α and some parameter which separates the creeds of the particular party voters H_α . Analogously, a negative D_F can be considered as a measure of discouragement to take part in public life and on the other hand, a positive $D_{L(C)}$ is a measure of citizen participation. In Fig.8 the Q -dependences of index V_D of the symmetric ($K_L = K_C = 0.5, D_F = -1.1, D_{L(C)} = 0$) and nonsymmetric ($K_L = 0.5, K_C = 0.4, D_F = -1.1, D_{L(C)} = 0$) models considered above are compared with the results for the model with $K_F = 1$ and positive $D_{L(C)} = 0.5$. As one would expect in the latter case the range of Q in which V_D changes ever so slightly is much broader.

V. SUMMARY

It is unlikely that a simple statistical physics model could be used to predict a social event, although certain sociophysicists believe that it is possible in some cases and for example Serge Galam³ claims "I do not think history could be predicted even in principle, given our current tools of research and perception of the world", however, at the same time he expresses a hope that "sociophysics in the future may yield real predictive tools". Anyway, it seems that sociophysics models can be successfully used to describe, explain and point out general

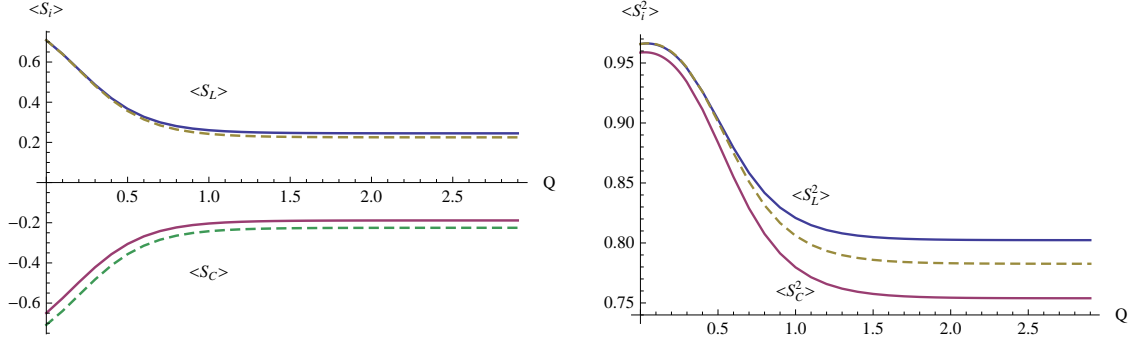


FIG. 5: Infinite chains: Magnetizations $\langle S_\alpha \rangle$ and $\langle S_\alpha^2 \rangle$ for $K_L = 0.5, K_C = 0.48$ (solid lines) and for $K_L = K_C = 0.5$ (dashed lines) at $T = 0.25$

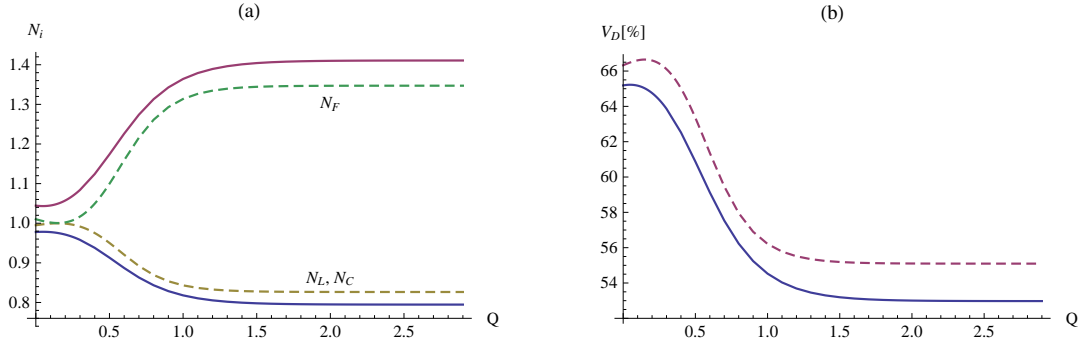


FIG. 6: Voter numbers N_α (a) and V_D -index (b) as functions of Q for $K_L = K_C = 0.5$ at $T = 0.25$ (solid lines) and $T = 0.2$ (dashed lines)

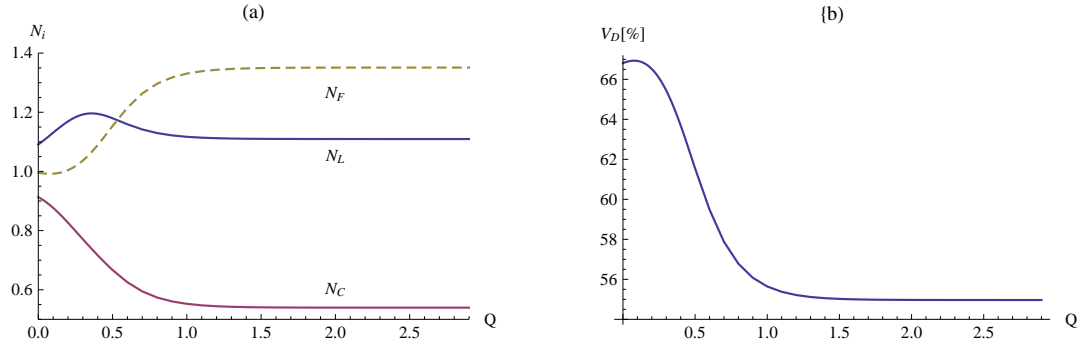


FIG. 7: Voter numbers N_α (a) and V_D -index (b) as functions of Q for nonsymmetric model with $K_L = 0.5, K_C = 0.4, K_F = 1$ at $T = 0.2$.

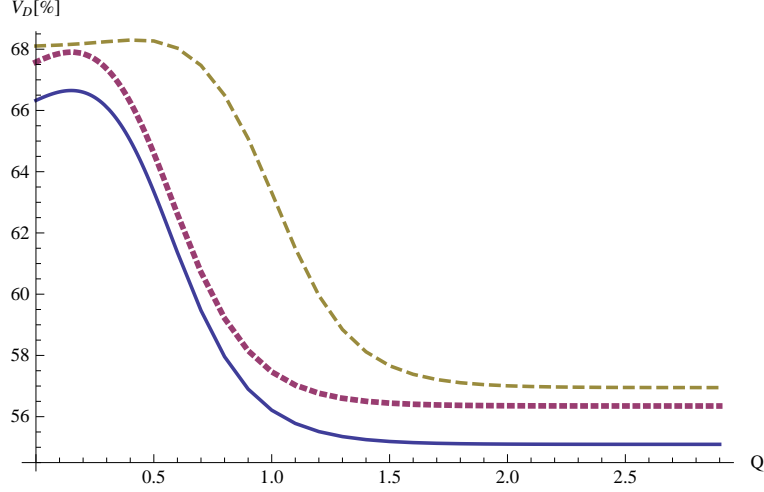


FIG. 8: V_D index as functions of Q for three models: symmetric $K_L = K_C = 0.5, K_F = 0$ (solid line), nonsymmetric with $K_L = 0.5, K_C = 0.4, K_F = 1$ (dotted), and nonsymmetric with $K_L = 0.5, K_C = 0.4, K_F = 1, D_L = D_C = 0.5$ (dashed) at $T = 0.2$.

features of social behavior.

In this paper to describe an influence of the social interplay between electorates of the two major parties, embodied by the coupling Q , on the quality of democracy, we propose the three-state Ising-like statistical physics model. The minimal number of parameters which define the model is three: the measure of the unity views of the two major parties (L, C) voters - $k = -K_L/T = -K_C/T$, the field which differentiates the creeds of the particular party voters - $h = -H_L/T = -H_C/T$, and the measure of a discouragement to take part in the public life of the F group citizens - $d = -D_F/T$. The measure of the democracy quality is V_D , index defined as a percentage of the total population that actually voted for two major parties in a given election. This index reflects not only rights but also the inclination of the citizens to participate in decision making, even if theoretically, which can be treated as an essence of democracy. To check a universality of the results we have applied three sets of the original parameters: (i) symmetric model ($K_L = K_C = 0.5, K_F = 0, D_F = -1.1, D_{L(C)} = 0$), (ii) nonsymmetric model ($K_L = 0.5, K_C = 0.4$, other as above), and (iii) ($K_L = 0.5, K_C = 0.4, K_F = 1, D_{L(C)} = 0.5$, other as above). In all cases, there is a range of Q in which the index V_D changes slightly, first increasing with Q , passes a maximum then at some characteristic point Q_f starts to fall rapidly, and at Q_c reaches a constant value (Fig.8). At the same time for the symmetric model (i) the numbers of both major

parties voters firstly slightly increase with increasing Q and then sharply decrease (Fig.6). For the nonsymmetric case ($K_L > K_C$), only the number of L party voters increases, reaches a maximum and then drops to some constant value whereas the number of C party voters decreases immediately with increasing of Q (Fig.7). When the value of both Q_f , Q_c , and the location of the V_D maximum depend on the model parameters, a collapse of V_D seems to be a general feature of the present model.

We conclude from the model that in the two party political system a reasonable conflict level between the electorates of the two major parties can be mutually beneficial for both parties and what is more for the quality of democracy measured by the index V_D . However, for the higher conflict level (higher degree of polarization), citizen participation decreases rapidly. For $Q > Q_c$ only so called hard or fixed electorates of the major parties want in public life. High percentage, and in the extreme case most of the society, decline voting for a party which can win a majority in the legislature ergo decline participation in a real decision making, which in fact means the collapse of the high quality democracy.

VI. APPENDIX

A. Decimation transformation parameters

The renormalized parameters (13) as functions of the original interactions $k_\alpha, h_\alpha, d_\alpha$ (RG recursion relations).

$$\begin{aligned} z_\alpha &= \lambda_0^\alpha, \quad h'_\alpha = \lambda_1^\alpha - \lambda_2^\alpha, \quad k'_\alpha = \frac{1}{4}(-2\lambda_3^\alpha + \lambda_4^\alpha + \lambda_5^\alpha), \quad d'_\alpha = -2\lambda_0^\alpha + \lambda_1^\alpha + \lambda_2^\alpha, \\ j'_\alpha &= \frac{1}{4}(-2\lambda_1^\alpha + 2\lambda_2^\alpha + \lambda_4^\alpha + \lambda_5^\alpha), \quad q'_\alpha = \frac{1}{4}(4\lambda_0^\alpha - 4\lambda_1^\alpha - 4\lambda_2^\alpha + 2\lambda_3^\alpha + \lambda_4^\alpha + \lambda_5^\alpha). \end{aligned} \quad (23)$$

$$\lambda_i^\alpha = \ln f_i^\alpha, \quad \omega_i^\alpha = \frac{1}{f_i^\alpha}, \quad i = 0, 1, \dots, 5, \quad \alpha = L, C, F. \quad (24)$$

$$\begin{aligned}
f_0^\alpha &= 1 + e^{d_\alpha - h_\alpha} + e^{d_\alpha + h_\alpha}, \\
f_1^\alpha &= e^{\frac{1}{2}(d_\alpha - h_\alpha - 2k_\alpha)}(e^{d_\alpha + q_\alpha} + e^{h_\alpha + k_\alpha} + e^{d_\alpha + 2h_\alpha + q_\alpha + 2k_\alpha + 2j_\alpha}), \\
f_2^\alpha &= e^{\frac{1}{2}(d_\alpha - 3h_\alpha + 2k_\alpha + 4j_\alpha)}(e^{d_\alpha + q_\alpha + 2k_\alpha} + e^{d_\alpha + 2h_\alpha + q_\alpha + 2j_\alpha} + e^{h_\alpha + k_\alpha + 2j_\alpha}), \\
f_3^\alpha &= e^{d_\alpha}(1 + e^{d_\alpha - h_\alpha + 2q_\alpha - 2j_\alpha} + e^{d_\alpha + h_\alpha + 2(q_\alpha + j_\alpha)}), \\
f_4^\alpha &= e^{d_\alpha}(e^{h_\alpha} + e^{d_\alpha + 2q_\alpha - 2k_\alpha} + e^{d_\alpha + 2(h_\alpha + q_\alpha + k_\alpha + 2j_\alpha)}), \\
f_5^\alpha &= e^{d_\alpha}(e^{-h_\alpha} + e^{d_\alpha + 2q_\alpha - 2k_\alpha} + e^{d_\alpha + 2(-h_\alpha + q_\alpha + k_\alpha - 2j_\alpha)}).
\end{aligned} \tag{25}$$

B. Single chain averages

$$\begin{aligned}
G_0^\alpha &= c_p^\alpha g_0^\alpha, \quad G_1^\alpha = (c_p^\alpha + c_d^\alpha)g_1^\alpha + c_h^\alpha(g_0^\alpha + g_2^\alpha), \quad G_2^\alpha = c_h^\alpha g_1^\alpha + c_p^\alpha g_2^\alpha + c_d^\alpha(g_0^\alpha + g_2^\alpha), \\
G_g^\alpha &= 2(c_q^\alpha + c_h^\alpha)(g_1^\alpha + g_3^\alpha) + c_k^\alpha(g_0^\alpha + 2g_2^\alpha + g_4^\alpha) + (2c_d^\alpha + c_j^\alpha + c_p^\alpha)g_g^\alpha, \\
G_3^\alpha &= (c_j^\alpha + c_k^\alpha)g_1^\alpha + c_h^\alpha(g_2^\alpha + g_4^\alpha + g_g^\alpha) + (c_j^\alpha + cK -^\alpha + c_p^\alpha)g_3^\alpha + c_d^\alpha(g_1^\alpha + 2g_3^\alpha) \\
&\quad + c_q^\alpha(g_0^\alpha + 2g_2^\alpha + g_4^\alpha + g_g^\alpha), \\
G_4^\alpha &= 2c_d^\alpha(g_2^\alpha + g_4^\alpha) + 2c_h^\alpha g_3^\alpha + 2c_q^\alpha(g_1^\alpha + g_3^\alpha) + c_p^\alpha g_4^\alpha + c_j^\alpha(g_0^\alpha + 2g_2^\alpha + g_4^\alpha) + c_k^\alpha g_g^\alpha.
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
c_p^\alpha &= \omega_0^\alpha, \quad c_h^\alpha = \frac{1}{2}(\omega_1^\alpha - \omega_2^\alpha), \quad c_k^\alpha = \frac{1}{4}(\omega_5^\alpha + \omega_4^\alpha - 2\omega_3^\alpha), \quad c_d^\alpha = \frac{1}{2}(\omega_1^\alpha + \omega_2^\alpha) - \omega_0^\alpha, \\
c_q^\alpha &= \frac{1}{2}(\omega_2^\alpha - \omega_1^\alpha + \omega_4^\alpha - \omega_5^\alpha), \quad c_j^\alpha = \omega_0^\alpha - \omega_1^\alpha - \omega_2^\alpha + \frac{1}{2}(\omega_3^\alpha + \omega_4^\alpha + \omega_5^\alpha).
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
g_0^\alpha &= Tr_S S_2^\alpha [1 - (S_1^\alpha)^2][1 - (S_3^\alpha)^2]e^{H_0^\alpha}, \quad g_1^\alpha = \frac{1}{2}Tr_S S_2^\alpha S_1^\alpha [1 - (S_3^\alpha)^2]e^{H_0^\alpha}, \\
g_g^\alpha &= \frac{1}{4}Tr_S S_2^\alpha S_1^\alpha S_2^\alpha e^{H_0^\alpha}, \quad g_2^\alpha = Tr_S S_2^\alpha [-1 + \frac{3}{2}(S_1^\alpha)^2][1 - (S_3^\alpha)^2]e^{H_0^\alpha}, \\
g_3^\alpha &= \frac{1}{2}Tr_S S_2^\alpha S_3^\alpha [-1 + \frac{3}{2}(S_1^\alpha)^2]e^{H_0^\alpha}, \quad g_4^\alpha = Tr_S S_2^\alpha [-1 + \frac{3}{2}(S_1^\alpha)^2][-1 + \frac{3}{2}(S_3^\alpha)^2]e^{H_0^\alpha}.
\end{aligned} \tag{28}$$

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